Dealing with post-treatment complications using principal stratification

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1. Outline of the talk

- Introducing principal stratification as a tool to represent post-treatment complications within the Rubin causal model
- Examples of post-treatment complications
- Focus on a specific complication: nonignorable nonresponse on an outcome variable
- Definition of causal estimands and identification results in the presence of an instrumental variable
- Empirical example: evaluating the effects of financial aids to firms

2. Principal stratification in experimental and observational studies

- "Endogenous" selection problems: e.g., treatment selection on unobservables, non response, censoring or truncation "due to death"
- Post-treatment complications requiring to control for them $\longrightarrow$ the use of conditioning is improper
- "Potential outcomes" approach (Rubin, 1974): unit i ($i = 1, \ldots, N$) assigned to treatment t ($t = 0, 1$); $S_i(t)$ is potential indicator of a specific post-treatment event; $Y_i(t)$ is the potential outcome variable
- SUTVA assumed
- Causal effect defined as a comparison of $Y_i(0)$ with $Y_i(1)$

- We assume unconfoundedness given X (vector of pretreatment variables)

$$Ass. 1 : T \perp\!\!\!\perp S(0), S(1), Y(0), Y(1) | X$$

- The observed data are

$$(t_i, S(t_i), Y(t_i), x_i) \qquad i = 1, \ldots, N$$

- Within each cell defined by a specific value of the pre-treatment variables, the units can be stratified into (four) groups according to the joint value of the potential outcomes $(S_i(0), S_i(1))$ (Frangakis and Rubin, 2002)

$$\begin{aligned} 11 &= \{i : S_i(1) = S_i(0) = 1\} \\ 10 &= \{i : S_i(1) = 1, S_i(0) = 0\} \\ 01 &= \{i : S_i(1) = 0, S_i(0) = 1\} \\ 00 &= \{i : S_i(1) = S_i(0) = 0\} \end{aligned}$$

- Basic principal stratification P_0 with respect S is defined as the partition of units $i = 1, \cdots, n$ such that, within any set of P_0, all units have the same vector of $(S_i(0), S_i(1))$
- The principal stratum membership $G_i = \{11, 10, 01, 00\}$ for subject i is not affected by treatment assignment t_i, so it only reflects characteristics of subject i, and can be regarded as a covariate, which is only partially observed in the sample (Angrist et al., 1996)
- $\pi_{11|x}$, $\pi_{10|x}$, $\pi_{01|x}$, and $\pi_{00|x} = 1 - \pi_{11|x} - \pi_{10|x} - \pi_{01|x}$ are the population proportions of units belonging to each stratum in the cell $X = x$

- By unconfoundedness G_i is guaranteed to have the same distribution in both treatment arms, within cells defined by pre-treatment variables
- Usually, information on causal effects is contained in a particular principal stratum: a principal causal effect is a properly defined causal effect, because it is obtained as a comparison of $Y(0)$ and $Y(1)$ on a common set of units
- Assumption 1 (unconfoundedness) implies that $Y(0), Y(1) \perp\!\!\!\perp T | S(0), S(1), X \rightarrow$ treated and control units can be compared conditional on a principal stratum
- Principal strata play a similar role of control functions in structural equation models for deriving independence conditions

3. Post-treatment complications 1

- Non compliance (Angrist *et al.*, 1996):
  - T treatment assignment, S treatment received
  - under some identifying assumptions, information can be obtained on the effect on the principal stratum of compliers: $E(Y(1) - Y(0)|S(0) = 0, S(1) = 1)$
- Direct and indirect effects (Mealli and Rubin, 2003; Flores and Flores-Lagunes, 2007)
  - T treatment, S intermediate variable
  - evidence on the direct effect, not "mediated" by S, can be found in principal strata where $S(0) = S(1)$

4. Post-treatment complications 2

- Censoring due to death: quality of life (Rubin, 2006; Mattei and Mealli, 2007)
  - T treatment, S survival, Y quality of life
  - effect of T on Y should be sought for the "always survivors": $E(Y(1) - Y(0)|S(0) = 1, S(1) = 1)$
- Censoring due to death: wages (Zhang *et al.*, 2007)
  - T treatment, S employment, Y wages
  - effect of T on Y should be sought for the "always employed": $E(Y(1) - Y(0)|S(0) = 1, S(1) = 1)$
- Non response (Mealli and Pacini, 2008)
  - T treatment, S response
  - different type of censoring: Y existing but not observed for nonrespondents

5. Motivation and aims of the paper

- Focus on a specific post-treatment complication within a causal inference framework: nonignorable nonresponse on an outcome variable
- Motivating example: evaluation study in the field of financial aids to firms, where typically missingness on variables related to firms'performances can rarely be assumed missing at random
- By exploiting Principal Stratification, we propose identification strategies with an *instrumental* variable for nonresponse
- We focus on the different role and meaning of the instrumental variable
- We compare our framework with a general nonseparable selection model setting

6. Nonignorable nonresponse on the outcome

- Let T be a binary treatment which represents public financial assistance to firms ($T = 1$ for treatment and $T = 0$ for no treatment)
- T is assumed unconfounded given a vector of pre-treatment covariate X
- The intermediate post-treatment variable S represents the response to a post-treatment questionnaire on firms' performances, the outcome variable of interest being the turnover Y
- We are facing the post-treatment complication of a potentially nonignorable missing mechanism of the outcome variable: missingness on turnover (sales proceeds) variables can rarely be assumed missing at random

In order to highlight the identification issue, it is useful to state the correspondence between observed groups, defined by T and S, and latent strata:

Observed subgroups $O(T,S)$	Turnover Y	Latent strata
$O(1,1) = \{i : T_i = 1, S_i = 1\}$	OBS	11 or 10
$O(1,0) = \{i : T_i = 1, S_i = 0\}$	$\cdot$	00 or 01
$O(0,1) = \{i : T_i = 0, S_i = 1\}$	OBS	11 or 01
$O(0,0) = \{i : T_i = 0, S_i = 0\}$	$\cdot$	10 or 00

- All the four observed groups result from a mixture of two principal strata
- Observed proportions and principal strata proportions:

$$\hat{p}(S_i = 1 | T_i = 1) = \pi_{11} + \pi_{10}$$

$$\hat{p}(S_i = 1 | T_i = 0) = \pi_{11} + \pi_{01}$$

- Observed averages and principal strata mean outcomes:

$$\hat{Y}(O(1,1)) = \frac{\mu_{Y(1)_{11}} \cdot \pi_{11} + \mu_{Y(1)_{10}} \cdot \pi_{10}}{\pi_{11} + \pi_{10}}$$

$$\hat{Y}(O(0,1)) = \frac{\mu_{Y(0)_{11}} \cdot \pi_{11} + \mu_{Y(0)_{01}} \cdot \pi_{01}}{\pi_{11} + \pi_{01}}$$

- It is not possible to point-identify the strata proportions, as well as the distribution of Y within the strata, that would allow to estimate causal effects
- Common assumption used to improve identification: **monotonicity assumption** $\longrightarrow$ nonexistence of the 01stratum
- Monotonicity allows to identify strata proportions but not $\mu_{Y(1)_{11}}$ $\longrightarrow$ distributional assumptions would be required (unless an instrument is available)

7. Introducing an instrument for nonresponse

- The lack of nonparametric identification can be solved by introducing some exclusion restriction
- We consider the availability of an instrumental variable Z that can be regarded as :
  - an additional intervention or
  - an additional post-treatment variable

- Suppose that units are exposed to an additional treatment, related to nonresponse S but unrelated to the outcome Y
- Our example: several persons with a different job task may respond to the phone interview
- Define Z the indicator variable which assumes value 1 if an employee responds and and 0 if the manager responds
- With two binary treatments, T and Z, four potential outcomes can be defined for each post-treatment variable:

  $S(t, z), Y(t, z)$ for $t = 0, 1$ and $z = 0, 1$

- The following hypotheses hold:

  Assumption 2: unconfoundedness of T

  $$T \perp\!\!\!\perp S(0,0), S(0,1), S(1,0), S(1,1), Y(0,0), Y(0,1), Y(1,0), Y(1,1)$$

  Assumption 3: unconfoundedness of Z

  $$Z \perp\!\!\!\perp S(0,0), S(0,1), S(1,0), S(1,1), Y(0,0), Y(0,1), Y(1,0), Y(1,1)$$

- In order to characterize Z as an instrument, we impose the following exclusion-restriction type of assumption:

  Assumption 4: exclusion restriction

  $$Y(0,0) = Y(0,1) \;\text{ and }\; Y(1,0) = Y(1,1)$$

  i.e., the value assumed by the instrument is unrelated to the outcome

Principal strata with two binary treatments and a binary intermediate variable

G	S(0,0)	S(0,1)	S(1,0)	S(1,1)
1	0	0	0	0
2	0	0	0	1
3	0	0	1	0
4	0	0	1	1
5	0	1	0	0
6	0	1	0	1
7	0	1	1	0
8	0	1	1	1
9	1	0	0	0
10	1	0	0	1
11	1	0	1	0
12	1	0	1	1
13	1	1	0	0
14	1	1	0	1
15	1	1	1	0
16	1	1	1	1

How can the presence of an instrument be exploited to achieve identification of some causal estimands?

- Assume that the instrument is *perfect*:

$$Assumption\ \ 5:\ S(0,1) = 1 \text{ and } \mathrm{S}(1,1) = 1.$$

G	S(0,0)	S(0,1)	S(1,0)	S(1,1)
6	0	1	0	1
8	0	1	1	1
14	1	1	0	1
16	1	1	1	1

This assumption allows to point identify the marginal distribution of $Y(T = 0)$ and $Y(T = 1)$, so that estimands involving only these marginals, as the ATE (Average Treatment Effect), can be identified

8. Structural assumptions

Alternative identification assumptions (with respect to the *perfect* instrument) can be stated as forms of monotonicity of S:

$$Assumption\ \ 6:\ \ S(t,0) \leq S(t,1)\ \ \forall t$$

$$Assumption\ \ 7:\ \ S(0,z) \leq S(1,z)\ \ \forall z.$$

- Assumption 6 relates to the response behaviour w.r.t. the instrument
- Assumption 7 relates to the response behaviour w.r.t. the treatment

G	S(0,0)	S(0,1)	S(1,0)	S(1,1)
1	0	0	0	0
2	0	0	0	1
4	0	0	1	1
6	0	1	0	1
8	0	1	1	1
16	1	1	1	1

- Additional assumptions are required:

  $Assumption\ \ 8:\ \ \text{Stratum 0001 does not exists}$

  $Assumption\ \ 9:$ type of latent ignorability

  $$Y(1,0) \perp\!\!\!\perp S(1,0) | S(0,0)=0, S(0,1)=1, S(1,1)=1$$

- An alternative to Assumption 8 could be to introduce a *natural* ordering of the strata and eliminate the strata contradicting the order: for example strata 4 and 6 could not simultaneously exist

- Under SUTVA, unconfoundedness of T and Z, exclusion restriction, monotonicity w.r.t. T and Z and Ass. 8 and 9 we show that one can identify and estimate the causal effect of T for the subset of units reacting to the instrument under treatment and/or control (strata 6 and 8)
- $\hat{E}[Y(T=1) - Y(T=0) | G \in \{6,8\}] =$

$$\frac{\hat{\bar{Y}}(1,1|S=1)\cdot(\hat{\pi}_4+\hat{\pi}_6+\hat{\pi}_8+\hat{\pi}_{16}) - \hat{\bar{Y}}(1,0|S=1)\cdot(\hat{\pi}_4+\hat{\pi}_8+\hat{\pi}_{16})}{\hat{\pi}_6}$$

$$-\frac{\hat{\bar{Y}}(0,1|S=1)\cdot(\hat{\pi}_6+\hat{\pi}_8+\hat{\pi}_{16}) - \hat{\bar{Y}}(0,0|S=1)\cdot\hat{\pi}_{16}}{\hat{\pi}_6+\hat{\pi}_8}$$

- Under SUTVA, unconfoundedness of T and Z, exclusion restriction, monotonicity w.r.t. T and Z, supposing that Stratum 0101 does not exist and formulating Assumption 9 as $Y(1,0) \perp\!\!\!\perp S(1,0) | S(0,0)=0, S(0,1)=0, S(1,1)=1$ one can identify and estimate the causal effect of T for the always respondents and the units reacting to the instrument under control (strata 8 and 16)
- $\hat{E}[Y(T=1)-Y(T=0)|G \in \{8,16\}] =$

$$\frac{1}{\hat{\pi}_8+\hat{\pi}_{16}} \cdot \Big[\hat{\bar{Y}}(1,0|S=1) \cdot (\hat{\pi}_4+\hat{\pi}_8+\hat{\pi}_{16}) -$$

$$[\hat{\bar{Y}}(1,1|S=1) \cdot (\hat{\pi}_2+\hat{\pi}_4+\hat{\pi}_8+\hat{\pi}_{16}) - \hat{\bar{Y}}(1,0|S=1) \cdot (\hat{\pi}_4+\hat{\pi}_8+\hat{\pi}_{16})] \cdot \frac{\hat{\pi}_4}{\hat{\pi}_2}\Big]$$

$$-\hat{\bar{Y}}(0,1|S=1)$$

- Under SUTVA, unconfoundedness of T and Z, exclusion restriction, monotonicity w.r.t. T and Z, supposing that Stratum 0011 does not exist and formulating Assumption 9 as $Y(0,0) \perp\!\!\!\perp S(0,0) | S(1,0) = 1, S(0,1) = 1, S(1,1) = 1$ identification is again achieved for the causal effect of T for strata 8 and 16, but the estimator in this case simply:

$$\hat{\bar{Y}}(1,0|S=1) - \hat{\bar{Y}}(0,0|S=1)$$

9. Alternative assumptions for the instrument

- Consider now Z as an additional post-treatment variable, that precedes nonresponse and can be added to the other potential outcome variables
- Two potential outcomes can be defined for each post-treatment variable, Z, S, and Y: $Z(t)$, $S(t)$, $Y(t)$ for $t = 0, 1$
- The treatment is assumed randomized conditional on a set of pre-treatment covariates:

$$Assumption\ \ 10 : T \perp\!\!\!\perp Z(0), Z(1), S(0), S(1), Y(0), Y(1)$$

- In order to characterize Z as an instrument, we impose the following exclusion-restriction type of assumption:

$$Assumption\ \ 11 : \ Y(0) \perp\!\!\!\perp Z(0) \quad \text{and} \quad Y(1) \perp\!\!\!\perp Z(1)$$

Principal strata with one binary treatments and two binary intermediate variables

G	Z(0)	Z(1)	S(0)	S(1)
1	0	0	0	0
2	0	0	0	1
3	0	0	1	0
4	0	0	1	1
5	0	1	0	0
6	0	1	0	1
7	0	1	1	0
8	0	1	1	1
9	1	0	0	0
10	1	0	0	1
11	1	0	1	0
12	1	0	1	1
13	1	1	0	0
14	1	1	0	1
15	1	1	1	0
16	1	1	1	1

- Some identifying assumptions are required in order to exploit the information gathered from the instrument
- Monotonicity hypothesis of the instrument with respect to the treatment (which eliminates strata 9 to 12):

$$Assumption\;\; 12: \;\; Z(0) \leq Z(1)$$

- A type of latent ignorability (within strata defined only by the joint values of the two potential outcomes $Z(0)$ and $Z(1)$, nonresponse is ignorable):

$$Assumption\;\; 13:$$
$$Y(0) \perp\!\!\!\perp S(0) | Z(0)=k, Z(1)=h \;\; \forall h,k;$$
$$Y(1) \perp\!\!\!\perp S(1) | Z(0)=k, Z(1)=h \;\; \forall h,k$$

- We concentrate on the 3 latent strata defined by Z
- Consider the following equality:

$$f(Y(1)|Z(1)=0, S(1)=1) = f(Y(1)|Z(1)=0, Z(0)=0, S(1)=1)$$

which holds because Assumption 12 implies that if $Z(1) = 0$ then $Z(0) = 0$

- By Assumptions 13 and 11 we also have:

$$f(Y(1)|Z(1)=0, Z(0)=0, S(1)=1) = f(Y(1)|Z(1)=0, Z(0)=0)$$

$$= f(Y(1)|Z(1)=0) = f(Y(1))$$

- Respondents with $Z(1) = 0$ can be used to estimate $f(Y(1))$
- Analogously, respondents with $Z(0) = 1$ can be used to estimate $f(Y(0))$

$$\hat{E}[Y(1) - Y(0)] = \hat{\bar{Y}}(1|Z=0, S=1) - \hat{\bar{Y}}(0|Z=1, S=1)$$

10. Empirical illustration

- We consider the effects of interest-relief grants on investments (Programs for the Development of Crafts in Tuscany), delivered in years 2001 and 2002, on turnover in year 2005, for for small and medium handicraft enterprises
- We consider a subsample consisting of 101 treated firms and 101 controls
- There is some evidence that nonresponse may be nonignorable
- Instrument for nonresponse: Z assumes value 1 if an employee responds to the phone interview and 0 if the owner responds

Relevant sample quantities: Y is 2005 turnover in euros, T is the program indicator, Z is the instrumental variable and S is the response indicator

$\hat{\bar{Y}}(T=0\|S=1)$	414078
$\hat{\bar{Y}}(T=1\|S=1)$	451132
Diff	37054
$\hat{\bar{Y}}(T=0,Z=0\|S=1)$	362273
$\hat{\bar{Y}}(T=1,Z=0\|S=1)$	416327
$\hat{\bar{Y}}(T=0,Z=1\|S=1)$	676554
$\hat{\bar{Y}}(T=1,Z=1\|S=1)$	686734
$\hat{p}(S=1\|T=1,Z=1)$	0.722
$\hat{p}(S=1\|T=1,Z=0)$	0.710
$\hat{p}(S=1\|T=0,Z=1)$	0.714
$\hat{p}(S=1\|T=0,Z=0)$	0.628

Estimated strata proportions under different identifying assumptions

	$\pi_2 = 0$	$\pi_4 = 0$	$\pi_6 = 0$
$\hat{\pi}_1$	0.278	0.278	0.273
$\hat{\pi}_2$	0	0.008	0.013
$\hat{\pi}_4$	0.008	0	0
$\hat{\pi}_6$	0.013	0.005	0
$\hat{\pi}_8$	0.074	0.082	0.086
$\hat{\pi}_{16}$	0.628	0.628	0.628

We estimate the average effect in strata $G \in \{8, 16\}$ under the assumptions PS1-PS4, PS6-PS7, nonexistence of stratum 0011 and $Y(0,0) \perp\!\!\!\perp S(0,0) | S(1,0) = 1, S(0,1) = 1, S(1,1) = 1$:

$$\hat{\bar{Y}}(1,0|S=1) - \hat{\bar{Y}}(0,0|S=1) = 54054 \ \text{ euros}$$

11. Concluding remarks

- We have dealt with the problem of a nonignorable nonresponse on an outcome variable, on which a causal effect of a treatment is of interest
- Assumptions that characterize the instrument and allow identification of some causal estimands have been proposed, which are not the standard assumptions used in an IV setting with endogenuous regressors
- Results were derived within the PS framework, where the latent strata are generated by the primitive potential outcomes
- Identification strategies exploit the comparison between observed groups and latent groups (strata)

- Bounds for treatment effects
- Possible assumptions include:
  - reducing the number of strata
  - imposing certain features of the distribution of outcomes within or among strata: exclusion restrictions, stochastic dominance, various forms of ignorability and nonignorability for the selection mechanism

- Using PS, whatever the assumptions made, the result of inference is always a causal effect within one or more strata
- We cannot univocally identify the group the causal effect refers to, so we cannot univocally estimate the individual causal effects
- This issue also characterizes the Instrumental Variable literature where, under certain assumptions, only the effect on specific subpopulations can be identified
- This is a limitation created by the selection mechanism, rather than a drawback of the framework of principal stratification

12. General nonseparable selection model setting

- Consider the following structural model:

$$Y_i = g(T_i, \epsilon_i)$$

$$S_i = h(Z_i, T_i, \eta_i)$$

$$T_i = l(\omega_i); \quad Z_i = m(\nu_i)$$

where Y_i is observed only if $S_i = 1$ and under unconfoundedness: $\omega_i \perp\!\!\!\perp \eta_i, \epsilon_i$ and $\nu_i \perp\!\!\!\perp \eta_i, \epsilon_i$

- If we do not impose any functional restrictions on g, h, and l and we do not restrict ϵ_i, η_i, and ω_i to be scalars, potential outcomes can be retrieved as:

$$Y_i(t, z) = g(t, \epsilon_i)$$

$$S_i(t, z) = h(z, t, \eta_i)$$

for $t = \{0, 1\}$ and $z = \{0, 1\}$

- We can look for a function of η, $G(\eta)$, called *type of unit* (Imbens, 2006) such that $\epsilon \perp\!\!\!\perp S | G(\eta)$
- A natural choice for $G(.)$:

$$
\begin{array}{ll}
G(\eta) = G(\eta') & \text{if } \mathrm{h}(\mathrm{z}, \mathrm{t}, \eta) = \mathrm{h}(\mathrm{z}, \mathrm{t}, \eta') \ \ \forall \mathrm{z}, \mathrm{t} \\
G(\eta) \neq G(\eta') & \text{if } \mathrm{h}(\mathrm{z}, \mathrm{t}, \eta) \neq \mathrm{h}(\mathrm{z}, \mathrm{t}, \eta') \ \ \text{for some } \mathrm{z}, \mathrm{t}.
\end{array}
$$

  so that $\epsilon \perp\!\!\!\perp S | G(\eta)$ by construction
- This corresponds to finding a common set of units within the Rubin Causal Model, on which proper causal estimands can be defined
- Assumptions stated for principal stratification can be translated in assumptions for structural models, with or without a proper meaning in this context

13. Comparing assumptions

Assumption PS1	Assumption SM1
$T \perp\!\!\!\perp S(0), S(1), Y(0), Y(1) \mid X$	$\omega_i \perp\!\!\!\perp \eta_i, \epsilon_i$

Assumption PS2

$$T \perp\!\!\!\perp S(0,0), S(0,1), S(1,0), S(1,1), Y(0,0), Y(0,1), Y(1,0), Y(1,1)$$

Assumption SM2

$$T \perp\!\!\!\perp (\epsilon, \eta)$$

Assumption PS3

$$Z \perp\!\!\!\perp S(0,0), S(0,1), S(1,0), S(1,1), Y(0,0), Y(0,1), Y(1,0), Y(1,1)$$

Assumption PS4

$$Y(0,0) = Y(0,1) \text{ and } Y(1,0) = Y(1,1)$$

Assumption SM3-4

$$Z \perp\!\!\!\perp (\epsilon, \eta)$$

Assumption PS5 (perfect instrument)

$$S(0,1) = 1 \text{ and } S(1,1) = 1$$

Monotonicity of S

Assumption PS6	Assumption SM6
$S(t,0) \leq S(t,1) \;\; \forall t$	$h(t,0,\eta) \leq h(t,1,\eta) \;\; \forall t, \forall \eta$
Assumption PS7	**Assumption SM7**
$S(0,z) \leq S(1,z) \;\; \forall z$	$h(0,z,\eta) \leq h(1,z,\eta) \;\; \forall z, \forall \eta$

Additional identifying assumptions

Assumption PS8

Stratum 0001 does not exists

$\rightarrow$ No natural interpretation in SM: alternative assumption

h(t,z,η) $\leq h(t,z,\eta') \;\; \forall t, z, \;\; \eta \leq \eta'$

(Strata 0011 and 0101 cannot simultaneoulsy exist)

Assumption PS9

$$Y(1,0) \perp\!\!\!\perp S(1,0) | S(0,0) = 0, S(0,1) = 1, S(1,1) = 1$$

Z as an additional post-treatment variable

Assumption PS10 $T \perp\!\!\!\perp Z(0), Z(1), S(0), S(1), Y(0), Y(1)$	Assumption SM10 $T \perp\!\!\!\perp (\epsilon, \eta, \nu)$
Assumption PS11 $Y(0) \perp\!\!\!\perp Z(0)$ and $Y(1) \perp\!\!\!\perp Z(1)$	Assumption SM11 $Z \perp\!\!\!\perp (\epsilon)$
Assumption PS12 $Z(0) \leq Z(1)$	Assumption SM12 $m(0, \nu) \leq m(1, \nu) \ \forall \nu$

Assumption PS13

$$Y(0) \perp\!\!\!\perp S(0) | Z(0) = k, Z(1) = h \ \forall h, k; \ Y(1) \perp\!\!\!\perp S(1) | Z(0) = k, Z(1) = h \ \forall h, k$$

Assumption SM13

$$\epsilon \perp\!\!\!\perp S | G(\nu)$$

with $G(\nu)$ such that

$$G(\nu) = G(\nu') \quad \text{if } \mathrm{m}(\mathrm{t}, \nu) = \mathrm{m}(\mathrm{t}, \nu') \ \forall \mathrm{t}$$
$$G(\nu) \neq G(\nu') \quad \text{if } \mathrm{m}(\mathrm{t}, \nu) \neq \mathrm{m}(\mathrm{t}, \nu') \text{ for some t}$$